7.16: PROBLEM DEFINITION

Situation:
The velocity distribution in a pipe with turbulent flow is given by

$$\frac{V}{V_{\max}} = \left(\frac{y}{r_0}\right)^n$$

Find:
Derive a formula for α as a function of n.
Find α for $n = 1/7$.

SOLUTION

Flow rate equation

$$\begin{aligned}
\frac{V}{V_{\max}} &= \left(\frac{y}{r_0}\right)^n = \left(\frac{r_0 - r}{r_0}\right)^n = \left(1 - \frac{r}{r_0}\right)^n \\
Q &= \int_A V dA \\
&= \int_0^{r_0} V_{\max}\left(1 - \frac{r}{r_0}\right)^n 2\pi r dr \\
&= 2\pi V_{\max} \int_0^{r_0} \left(1 - \frac{r}{r_0}\right)^n r dr
\end{aligned}$$

Upon integration

$$Q = 2\pi V_{\max} r_0^2 \left[\left(\frac{1}{n+1}\right) - \left(\frac{1}{n+2}\right)\right]$$

Then

$$\begin{aligned}
\bar{V} &= Q/A = 2V_{\max}\left[\left(\frac{1}{n+1}\right) - \left(\frac{1}{n+2}\right)\right] \\
&= \frac{2V_{\max}}{(n+1)(n+2)}
\end{aligned}$$

Kinetic energy correction factor

$$\alpha = \frac{1}{A}\int_0^{r_0}\left[\frac{V_{\max}\left(1 - \frac{r}{r_0}\right)^n}{\frac{2V_{\max}}{(n+1)(n+2)}}\right]^3 2\pi r dr$$

Upon integration one gets

$$\boxed{\alpha = \frac{1}{4}\left[\frac{[(n+2)(n+1)]^3}{(3n+2)(3n+1)}\right]}$$

If $n = 1/7$, then

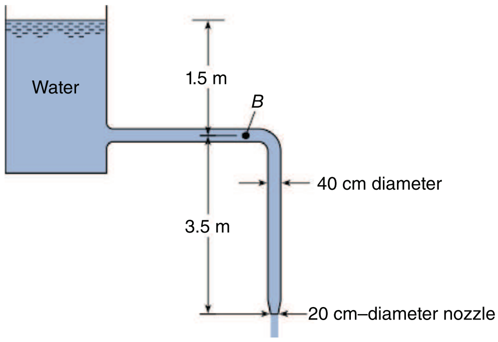
$$\alpha = \frac{1}{4}\left[\frac{\left[(\frac{1}{7}+2)(\frac{1}{7}+1)\right]^3}{\left(3\left(\frac{1}{7}\right)+2\right)\left(3\left(\frac{1}{7}\right)+1\right)}\right]$$

$$\boxed{\alpha = 1.06}$$

7.24: PROBLEM DEFINITION

Situation:

Water flowing from a tank into a pipe connected to a nozzle.
$\alpha = 1.0,\ D_B = 40\,\text{cm}.$
$D_0 = 20\,\text{cm},\ z_0 = 0\,\text{m}.$
$z_B = 3.5\,\text{m},\ z_r = 5\,\text{m}.$



Find:

(a) Discharge in pipe (m^3/s).
(b) Pressure at point B (kPa).

Assumptions:

$\gamma = 9810\,\text{N}/\,\text{m}^3.$

PLAN

1. Find velocity at nozzle by applying the energy equation.
2. Find discharge by applying $Q = A_o V_o$
3. Find the pressure by applying the energy equation.

SOLUTION

1. Energy equation (point 1 on reservoir surface, point 2 at outlet)

$$\begin{aligned}\frac{p_{\text{reser.}}}{\gamma} + \frac{V_r^2}{2g} + z_r &= \frac{p_{\text{outlet}}}{\gamma} + \frac{V_0^2}{2g} + z_0 \\ 0 + 0 + 5 &= 0 + \frac{V_0^2}{2g} \\ V_0 &= 9.90 \text{ m/s}\end{aligned}$$

2. Flow rate equation

$$\begin{aligned}Q &= V_0 A_0 \\ &= 9.90\,\text{m}/\,\text{s} \times (\frac{\pi}{4}) \times (0.20\,\text{m})^2 \\ &\boxed{Q = 0.311 \text{ m}^3/\text{s}}\end{aligned}$$

3. Energy equation (point 1 on reservoir surface, point 2 at B)

$$0 + 0 + 5 = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + 3.5$$

where

$$\begin{aligned} V_B &= \frac{Q}{V_B} = \frac{0.311\,\mathrm{m}^3/\,\mathrm{s}}{(\pi/4) \times (0.4\,\mathrm{m})^2} = 2.48\ \mathrm{m/s} \\ \frac{V_B^2}{2g} &= 0.312\,\mathrm{m} \end{aligned}$$

so :

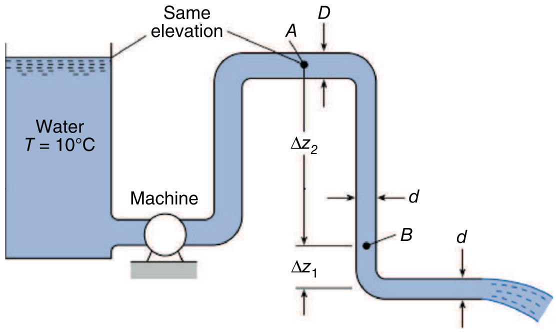
$$5 = \frac{p_B}{\gamma} + 0.312 + 3.5$$

$$\boxed{p_B = 11.6\ kPa}$$

7.40: PROBLEM DEFINITION

Situation:

A reservoir discharges water into a pipe with a machine.
$d = 15\,\text{cm}$, $D = 30\,\text{cm}$.
$\Delta z_1 = 1.8\,\text{m}$, $\Delta z_2 = 3.6\,\text{m}$.
$Q = 0.3\,\text{m}^3/\text{s}$.



Find:

Is the machine a pump or a turbine?
Pressures at points A and B (kPa).

Assumptions:

Machine is a pump, and then check for reasonable values.
$\alpha = 1.0$.

PLAN

Apply the energy equation between the top of the tank and the exit, then between point B and the exit, finally between point A and the exit.

SOLUTION

Energy equation between top of tank and exit:

$$z_1 + h_p = \frac{V_2^2}{2g} + z_2$$

Assuming the machine is a pump. If the machine is a turbine, then h_p will be negative. The velocity at the exit is

$$V_2 = \frac{Q}{A_2} = \frac{0.3\,\text{m}^3/\,\text{s}}{\frac{\pi}{4}\,(0.15\,\text{m})^2} = 17\text{ m/s}$$

Solving for h_p and taking the pipe exit as zero elevation we have

$$h_p = \frac{(17\,\text{m/s})^2}{2 \times 9.81\,\text{m/s}^2} + 3.6\,\text{m} - 1.8\,\text{m} = 9.3\text{ m}$$

Therefore the machine is a pump.
Applying the energy equation between point B and the exit gives

$$\frac{p_B}{\gamma} + z_B = z_2$$

Solving for p_B we have

$$\begin{aligned} p_B &= \gamma(z_2 - z_B) \\ p_B &= -1.8\,\text{m} \times 9810\,\text{N/m}^3 = \text{-}\,17.7\text{ kPa} \\ & \boxed{p_B = \text{-}17.7\text{ kPa}} \end{aligned}$$

Velocity at A

$$V_A = \left(\frac{15}{30}\right)^2 \times 17\,\text{m/s} = 4.25\text{ m/s}$$

Applying the energy equation between point A and the exit gives

$$\frac{p_A}{\gamma} + z_A + \frac{V_A^2}{2g} = \frac{V_2^2}{2g}$$

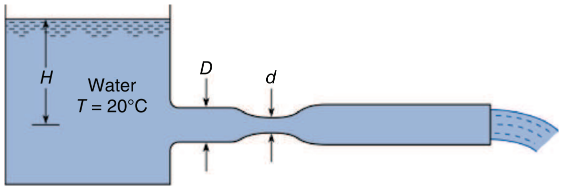
so

$$\begin{aligned} p_A &= \gamma\left(\frac{V_2^2}{2g} - z_A - \frac{V_A^2}{2g}\right) \\ &= 9810\,\text{N/m}^3 \times \left(\frac{(17\,\text{m/s})^2 - (4.25\,\text{m/s})^2}{2 \times 9.81\,\text{m/s}^2} - 5.4\right) \\ &= 82.5\text{ kPa} \\ & \boxed{p_A = 82.5\text{ kPa}} \end{aligned}$$

7.42: PROBLEM DEFINITION

Situation:

A reservoir discharges to a pipe with a venturi meter before draining to atmosphere.
$D = 35\,\text{cm}$, $d = 15\,\text{cm}$.
$p_{\text{atm}} = 100\,\text{kPa}$, $h_L = 1.5V_2^2/2g$.



Find:

Head at incipient cavitation (m).
Discharge at incipient cavitation (m^3/s).

Assumptions:

$\alpha = 1.0$.

Properties:

From Table A.5 $p_v = 2340$ Pa, abs.

PLAN

First apply the energy equation from the Venturi section to the end of the pipe. Then apply the energy equation from reservoir water surface to outlet:

SOLUTION

Energy equation from Venturi section to end of pipe:

$$
\begin{aligned}
\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L \\
\frac{p_{\text{vapor}}}{\gamma} + \frac{V_1^2}{2g} &= 0 + \frac{V_2^2}{2g} + 1.5\frac{V_2^2}{2g} \\
p_{\text{vapor}} &= 2,340 \text{ Pa abs.} = -97,660 \text{ Pa gage}
\end{aligned}
$$

Continuity principle

$$
\begin{aligned}
V_1 A_1 &= V_2 A_2 \\
V_1 &= \frac{V_2 A_2}{A_1} \\
&= 5.44 V_2
\end{aligned}
$$

Then

$$
\frac{V_1^2}{2g} = 29.64\frac{V_2^2}{2g}
$$

Substituting into energy equation

$$\begin{aligned} -97,660/9,790 + 29.64\frac{V_2^2}{2g} &= 2.5\frac{V_2^2}{2g} \\ V_2 &= 2.685 \text{ m/s} \end{aligned}$$

Flow rate equation

$$\begin{aligned} Q &= V_2 A_2 \\ &= 2.685\,\text{m/s} \times \pi/4 \times (0.35\,\text{m})^2 \\ &\boxed{Q = 0.258 \text{ m}^3/\text{s}} \end{aligned}$$

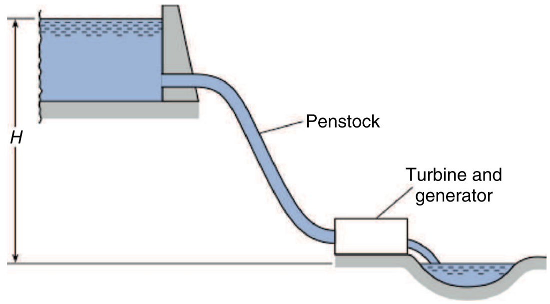
Energy equation from reservoir water surface to outlet:

$$\begin{aligned} H &= \frac{V_2^2}{2g} + h_L \\ H &= 2.5\frac{V_2^2}{2g} \\ &\boxed{H = 0.919 \text{ m}} \end{aligned}$$

7.57: PROBLEM DEFINITION

Situation:

An engineer is estimating the power that can be produced by a small stream.
$Q = 0.04\ \text{m}^3/\text{s}$, $T = 5\,^\circ\text{C}$, $H = 10\,\text{m}$.



Find:
Estimate the maximum power that can be generated (kW) if:
$h_L = 0\,\text{m}$, $\eta_\text{t} = 100\%$, $\eta_\text{g} = 100\%$.
$h_L = 1.7\,\text{m}$, $\eta_\text{t} = 70\%$, $\eta_\text{g} = 90\%$.

PLAN To find the head of the turbine (h_t), apply the energy equation from the upper water surface (section 1) to the lower water surface (section 2). To calculate power, use $P = \eta(\dot{m}gh_\text{t})$, where η accounts for the combined efficiency of the turbine and generator.

SOLUTION

Energy equation

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L \qquad (1)$$

Term by term analysis

$$\begin{aligned} p_1 &= 0; \quad V_1 \approx 0 \\ p_2 &= 0; \quad V_2 \approx 0 \\ z_1 - z_2 &= H \end{aligned}$$

Eq. (1) becomes

$$\begin{aligned} H &= h_\text{t} + h_L \\ h_\text{t} &= H - h_L \end{aligned}$$

Flow rate

$$\begin{aligned} \dot{m}g &= \gamma Q \\ &= \left(9810\,\mathrm{N/m^3}\right)\left(0.04\,\mathrm{m^3/\,s}\right) \\ &= 392.4\,\mathrm{N/s} \end{aligned}$$

Power (case a)

$$\begin{aligned} P &= \dot{m}gh_t \\ &= \dot{m}gH \\ &= (392.4\,\mathrm{N/s})\,(10\,\mathrm{m}) \\ &= 4.0\,\mathrm{kW} \end{aligned}$$

Power (case b).

$$\begin{aligned} P &= \eta\dot{m}g\,(H - h_L) \\ &= (0.7)(0.9)\,(392.4\,\mathrm{N/s})\,(10\,\mathrm{m} - 1.7\,\mathrm{m}) \\ &= 2.1\,\mathrm{kW} \end{aligned}$$

$$\boxed{\text{Power (case a)} = 4.0\ \text{kW}}$$

$$\boxed{\text{Power (case b)} = 2.1\ \text{kW}}$$

REVIEW

1. In the ideal case (case a), all of the elevation head is used to make power. When typical head losses and machine efficiencies are accounted for, the power production is cut by nearly 50%.

2. From Ohm's law, a power of 2.13 kW will produce a current of about 17.5 amps at a voltage of 120V. Thus, the turbine will provide enough power for about 1 typical household circuit. It is unlikely the turbine system will be practical (too expensive and not enough power for a homeowner).

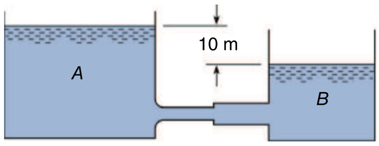
7.66: PROBLEM DEFINITION

Situation:

Two tanks are connected by a pipe with a sudden expansion.

$\Delta z = 10\,\text{m}$, $A_1 = 8\,\text{cm}^2$.

$A_2 = 25\,\text{cm}^2$.



Find:

Discharge between two tanks (m^3/s)

PLAN

Apply the energy equation from water surface in A to water surface in B.

SOLUTION

Energy equation (top of reservoir A to top of reservoir B)

$$\begin{aligned} \frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A &= \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + \Sigma h_L \\ 0 + 0 + 10\,\text{m} &= 0 + 0 + 0 + \Sigma h_L \end{aligned} \tag{1}$$

Let the pipe from A be called pipe 1. Let the pipe into B be called pipe 2 Then

$$\Sigma h_L = \frac{(V_1 - V_2)^2}{2g} + \frac{V_2^2}{2g} \tag{2}$$

Continuity principle:

$$\begin{aligned} V_1 A_1 &= V_2 A_2 \\ V_1 &= \frac{V_2 A_2}{A_1} = V_2 \frac{(25\,\text{cm}^2)}{(8\,\text{cm}^2)} = 3.125 V_2 \end{aligned} \tag{3}$$

Combine Eq. (1), (2), and (3):

$$\begin{aligned} 10\,\text{m} &= \frac{(3.125 V_2 - V_2)^2}{2\,(9.81\,\text{m/s}^2)} + \frac{{V_2}^2}{2\,(9.81\,\text{m/s}^2)} \\ V_2 &= 5.964\,\text{m/s} \end{aligned}$$

Flow rate equation:

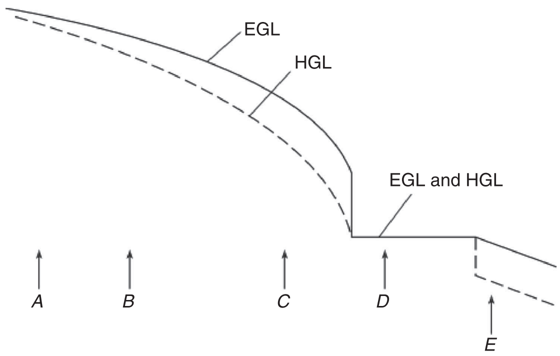
$$\begin{aligned} Q &= V_2 A_2 \\ &= (5.964\,\text{m/s})\,(25\,\text{cm}^2)\left(\frac{1.0\,\text{m}}{100\,\text{cm}}\right)^2 \end{aligned}$$

$$\boxed{Q = 0.0149\,\text{m}^3/\text{s}}$$

7.78: PROBLEM DEFINITION

Situation:

An HGL and EGL are shown for a flow system.

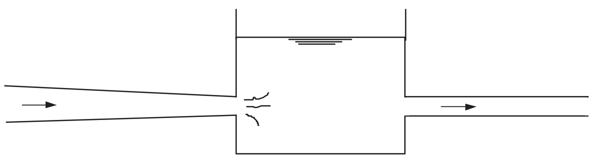


Find:

(a) Direction of flow.
(b) Whether there is a reservoir.
(c) Whether the diameter at E is uniform or variable.
(d) Whether there is a pump.
(e) Sketch a physical set up that could exist between C and D.
(f) Whether there is anything else revealed by the sketch.

SOLUTION

(a) Flow is from A to E because EGL slopes downward in that direction.
(b) Yes, at D, because EGL and HGL are coincident there.
(c) Uniform diameter because $V^2/2g$ is constant (EGL and HGL uniformly spaced).
(d) No, because EGL is always dropping (no energy added).
(e)

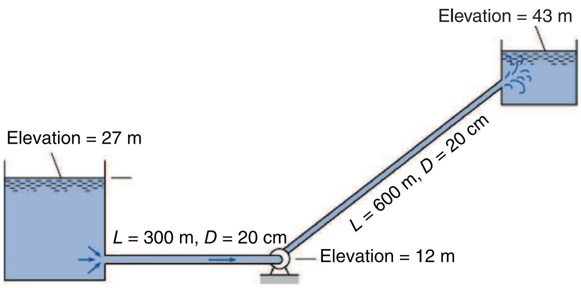


(f) Nothing else.

7.86: PROBLEM DEFINITION

Situation:

Water is pumped from a lower reservoir to an upper one.
$z_1 = 27\,\text{m}$, $z_2 = 43\,\text{m}$.
$L_1 = 300\,\text{m}$, $L_2 = 600\,\text{m}$.
$D_1 = 20\,\text{cm}$, $D_2 = 20\,\text{cm}$.
$Q = 0.1\,\text{m}^3/\,\text{s}$, $h_L = 0.018\frac{L}{D}\frac{V^2}{2g}$.



Find:

(a) Power supplied to the pump (kW).
(b) Sketch the HGL and EGL.

Properties:

Water (20 °C), Table A.5: $\gamma = 9810\,\text{N/m}^3$.

PLAN

Apply the flow rate equation to find the velocity. Then calculate head loss. Next apply the energy equation from water surface to water surface to find the head the pump provides. Finally, apply the power equation.

SOLUTION

Flow rate equation

$$\begin{aligned} V &= \frac{Q}{A} \\ &= \frac{0.1\,\text{m}^3/\,\text{s}}{(\pi/4) \times (0.2\,\text{m})^2} \\ &= 3.2\text{ m/s} \end{aligned}$$

Head loss

$$\begin{aligned} h_L &= \left(0.018\frac{L}{D}\frac{V^2}{2g}\right) + \left(\frac{V^2}{2g}\right) \\ &= 0.018\left(\frac{900\,\text{m}}{0.2\,\text{m}}\right)\frac{(3.2\,\text{m/s})^2}{2\,(9.81\,\text{m/s}^2)} + \frac{(3.2\,\text{m/s})^2}{2\,(9.81\,\text{m/s}^2)} \\ &= 42.64\,\text{m} \end{aligned}$$

Energy equation

$$
\begin{aligned}
\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_L \\
0 + 0 + 27 + h_p &= 0 + 0 + 43 + 42.64 \\
h_p &= 58.64 \text{ m}
\end{aligned}
$$

Power equation

$$
\begin{aligned}
P &= Q\gamma h_p \\
&= 0.1\,\text{m}^3/\,\text{s} \times 9810\,\text{N/m}^3 \times 58.64\,\text{m} \\
&= 57,526\,\text{N-m/s}
\end{aligned}
$$

$$
\boxed{P = 58\,\text{kW}}
$$